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ABSTRACT

Antenna patterns can be designed to have broad notches in regions where clutter and jamming are expected to be received. A number of narrowband pattern synthesis techniques exist to design notched antenna patterns, but they break down in the presence of wideband signals on arrays which employ a combination of phase and time steering. The authors derive definitions for wideband antenna patterns and present a new synthesis procedure which can be used to design notched patterns for signals having specified bandwidths on any type of array.

SECTION 1

INTRODUCTION

Antenna patterns designed to have broad notches can be utilized to reduce interference over wide angular regions [1]. The notched patterns, which are prestored, can be used on antennas which support independent amplitude and phase weighting at the element level. Implementation entails storing and downloading the amplitude and phase weights corresponding to the desired pattern.

The position and width of the notches will depend upon the application. A wide notch extending from the underside of the beam through the vertical line may be desirable in airborne radar applications to reduce ground clutter, while patterns having narrower notches should be useful in ground-based radar applications to lower horizon clutter and jamming. If the antenna is rectangular and of row-column construction, the composite pattern can be represented as the product of a row pattern with a column pattern. The separability implies that an elevation notch placed in the column pattern of a vertical rectangular array will automatically appear at all azimuths in the composite pattern.

The notches shift with the pattern as the beam pointing direction is changed. Thus, if it is desired to maintain a notch over an angular sector $\Delta\theta$ while the beam is scanned over a range of $\Delta\theta_s$, the angular extent of the notch must be at least $\Delta\theta + \Delta\theta_S$. The depth of the notch will be limited by systematic and random errors within the antenna. The notches will be most useful in the region near the mainlobe where the rms sidelobe level without a notch is usually well above the random error level. The notches will also cause some loss in power gain. The loss in gain increases as the notch is moved closer to the mainlobe and as the width and depth of the notch are increased.

A few authors have proposed narrowband synthesis techniques for designing notched antenna patterns [2,3]. The narrowband techniques can be used on arrays which employ only phase or only time steering, but break down when used with wide bandwidths on arrays which employ a combination of phase and time steering. Time steering is required with

wideband operation to prevent decorrelation of the signal across the array face. The cost of the time delay units, however, drives the designer of large arrays to a subarray architecture utilizing phase steering within subarrays and time steering between them.

We can obtain insight into the reason why narrowband synthesis fails on phase-time steered arrays by considering the pattern of a uniformly weighted rectangular array made up of M subarrays, each having N elements. The pattern can be decomposed into the product of a subarray pattern and an array factor. The array factor contains (N-1) grating lobes which line up with the nulls in the subarray pattern at the design frequency. As frequency is changed, grating lobes in the time steered array factor remain fixed in angle while the subarray pattern scans. Misalignment between subarray pattern nulls and the grating lobes allows the grating lobes to pop up. Since broadband patterns are obtained by integrating over frequency, the grating lobe will dominate the integrand. If the element weights are chosen to form a broad angular notch, integrating over the grating lobe will cause the notch to fill in.

One way to mitigate the problem of grating lobes filling in the notches in wideband operation is to increase the number of subarrays. The latter approach will reduce the number of grating lobes, but at the expense of increasing the cost and complexity of the beamforming and time delay steering hardware. The alternative approach, which the authors have taken, is to use wideband pattern synthesis techniques.

The mathematics governing the response of antennas to wideband signals is formulated in Section 2. Section 3 presents a new technique which can be used to design wideband notched antenna patterns. Computer simulation results are presented in Section 4 to verify the findings and the work is summarized in Section 5.

SECTION 2

ANTENNA RESPONSES TO WIDEBAND SIGNALS

A few authors have examined the response of antennas to wideband signals [4]. Let us first review the case where the antenna receives a wideband deterministic signal. Assuming that the antenna is a linear array, the power at the output of the receiver, due to a far-field point source, is given by

$$P_s = \left| \int_{-\infty}^{\infty} s(\omega) g(\mu, \omega) h(\omega) d\omega \right|^2, \tag{1}$$

where $s(\omega)$ equals the Fourier transform of a deterministic signal at radian frequency ω, $g(\mu, \omega)$ equals the Fourier transform of the impulse response of the antenna at an angle of θ, μ equals $\sin\theta$, and $h(\omega)$ is the receiver transfer function. If we assume the transform of the signal and the receiver transfer function are rectangular and centered at ω_o with radian bandwidth 2β, (1) simplifies to

$$\begin{aligned} P_s &= \frac{S}{2\beta} \left| \int_{\omega_0-\beta}^{\omega_0+\beta} g(\mu, \omega) d\omega \right|^2 \\ &= SG_s(\mu, \beta) \end{aligned} \tag{2}$$

where $|s(\omega)|^2$ has been assumed to be $S/2\beta$, $h(\omega)$ equals unity, and $G_s(\mu, \beta)$ is defined to be the wideband power pattern of a deterministic signal. The effect of antenna errors can usually be ignored as long as the signal is within the 3 dB beamwidth of the antenna.

We next consider the expected value of the output power due to a far-field wideband noise source j(t) received in the sidelobes. Since the interference is received in the sidelobes,

we can no longer ignore antenna errors. Both the noise source and the antenna pattern must be treated as random processes. The power out of the receiving antenna is given by

$$\langle P_n \rangle = \left\langle \left| \int_{-\infty}^{\infty} j(\omega) g(\mu, \omega) h(\omega) d\omega \right|^2 \right\rangle$$

$$= \int_{-\infty}^{\infty} \left\langle |j(\omega)|^2 \right\rangle \left\langle |g(\mu, \omega) h(\omega)|^2 \right\rangle d\omega \qquad (3)$$

where $\langle \ \rangle$ denotes expectation. The expectation on the interferer is over different random process waveforms while the expectation on the product of the antenna and receiver transfer functions is over different antennas. By bringing the expectation under the integral sign in (3), we have assumed that ensemble averaging over different antennas and integrating over frequency are interchangeable. If we assume that the receiver transfer function is rectangular and centered at ω_0 with bandwidth 2β, and further assume the noise to be white with spectral density $J/2\beta$, (3) simplifies to

$$\langle P_n \rangle = \frac{J}{2\beta} \int_{\omega_0-\beta}^{\omega_0+\beta} \left\langle |g(\mu, \omega)|^2 \right\rangle d\omega = J G_n(\mu, \beta), \qquad (4)$$

where we define $G_n(\mu, \beta)$ to be the wideband power pattern seen by a far-field white noise source.

SECTION 3

WIDEBAND PATTERN SYNTHESIS TECHNIQUE FOR DESIGNING NOTCHED ANTENNA PATTERNS

Shore [2] developed a narrowband synthesis procedure which can be used to design notched antenna patterns for phase steered arrays. His algorithm is based upon minimizing a weighted sum of the average CW power over the angular sector (2ε) to be nulled, plus the norm of a perturbation vector. The shape of the antenna pattern outside the notched region can be preserved at the expense of the pattern shape in the notched region by weighting the norm more heavily than the average power term. The perturbation vector is the difference between the quiescent element weight vector (w_0) and the desired weight vector (w) which produces the notched pattern. Mathematically,

$$P = \alpha_0 (w - w_0)^+ (w - w_0) + \frac{\alpha_1}{2\varepsilon} \int_{\mu_c - \varepsilon}^{\mu_c + \varepsilon} |g(\mu, \omega)|^2 d\mu \, , \tag{5}$$

where

α_0, α_1 = weighting constants

μ_c = $\sin\theta_c$ = center of notched region

$+$ = conjugate transpose

Shore's procedure can be modified to design broadband notched patterns for phase, time, or phase-time steered arrays having any bandwidth. The new procedure entails replacing the second term in (5) by

$$P_2 = \frac{\alpha_1}{4\varepsilon\beta} \int_{\omega_0 - \beta}^{\omega_0 + \beta} \int_{\mu_c - \varepsilon}^{\mu_c + \varepsilon} |g(\mu, \omega)|^2 d\mu d\omega \, , \tag{6}$$

Integrating (6), it can be shown that (see Appendix A)

$$P_2 = w^{+}Hw, \tag{7}$$

where H is a square matrix defined by (A-11) in Appendix A. The desired weight vector can be obtained by minimizing

$$P = \alpha_0(w - w_0)^{+}(w - w_0) + \alpha_1 w^{+}Hw. \tag{8}$$

Taking the derivative of P, with respect to w and equating to zero, we get

$$2\alpha_0(w - w_0) + 2\alpha_1 Hw = 0\,. \tag{9}$$

Solving for the new weight vector

$$w = [I + \alpha H]^{-1} w_0\,. \tag{10}$$

where α equals α_1/α_0, and I is an identity matrix. The relative emphasis assigned to minimizing the squared weight perturbations while simultaneously minimizing the average power in the sidelobe sector to be nulled is controlled by adjusting α. The ratio affects the depth of the notch, as well as the loss in gain, at the peak of the beam.

It is noteworthy that the procedure is easily extended to include multiple notches. The weight vector for the case of K notches is readily shown to be

$$w = \left[I + \sum_{k=1}^{K} \alpha'_k H_k\right]^{-1} w_0 \tag{11}$$

where

$\alpha'_k = \alpha_k/\alpha_0$

SECTION 4

COMPUTER SIMULATION

The wideband synthesis technique developed in Section 3 for designing notched antenna patterns was simulated on the computer. The linear array consisting of 96 isotropic elements spaced 0.58 wavelengths apart at the center frequency. A 27 dB, nbar 4, Taylor weighting was imposed across the aperture. The array consisted of 6 contiguous subarrays, each having 16 elements. Elements within each subarray were phase steered and time steering was used to align the subarrays to each other. The antenna pattern was described mathematically using Equation A-2. All bandwidths referred to in the figures have been normalized to f_0 and are expressed as percentages (i.e., $200\beta/f_0$).

Figure 1 shows the linear array patterns produced by the synthesis procedure described in Section 3 for different values of the emphasis parameter α. The prescribed notch is 10° wide and centered 12.5° from the beampointing direction. The weights were calculated using (10), with bandwidth set to zero when determining the H matrix, i.e., there was no integration over frequency and H was given by H_0 defined in (A-12).

Figures 2 and 3 show the effect of increasing the bandwidth of the far-field noise source while maintaining fixed narrowband (CW) element weights $(H = H_0)$. The patterns in Figure 2 were designed without a notch while those in Figure 3 had a 10° notch $(\alpha = 250)$. In both figures the sidelobe nulls are observed to fill in with increasing bandwidth. In Figure 3, a grating lobe is observed to pop up and fill in the 10° notched sector. The grating lobe can be attenuated using the wideband synthesis procedure to calculate the element weights (H given by A-11).

Figure 4 shows the same three cases presented in Figure 3 with wideband weights (2β set equal to bandwidth of source). The grating lobe within the notched region is suppressed at the expense of raising the grating lobes in other (don't care) regions of the pattern and incurring some loss in peak gain.

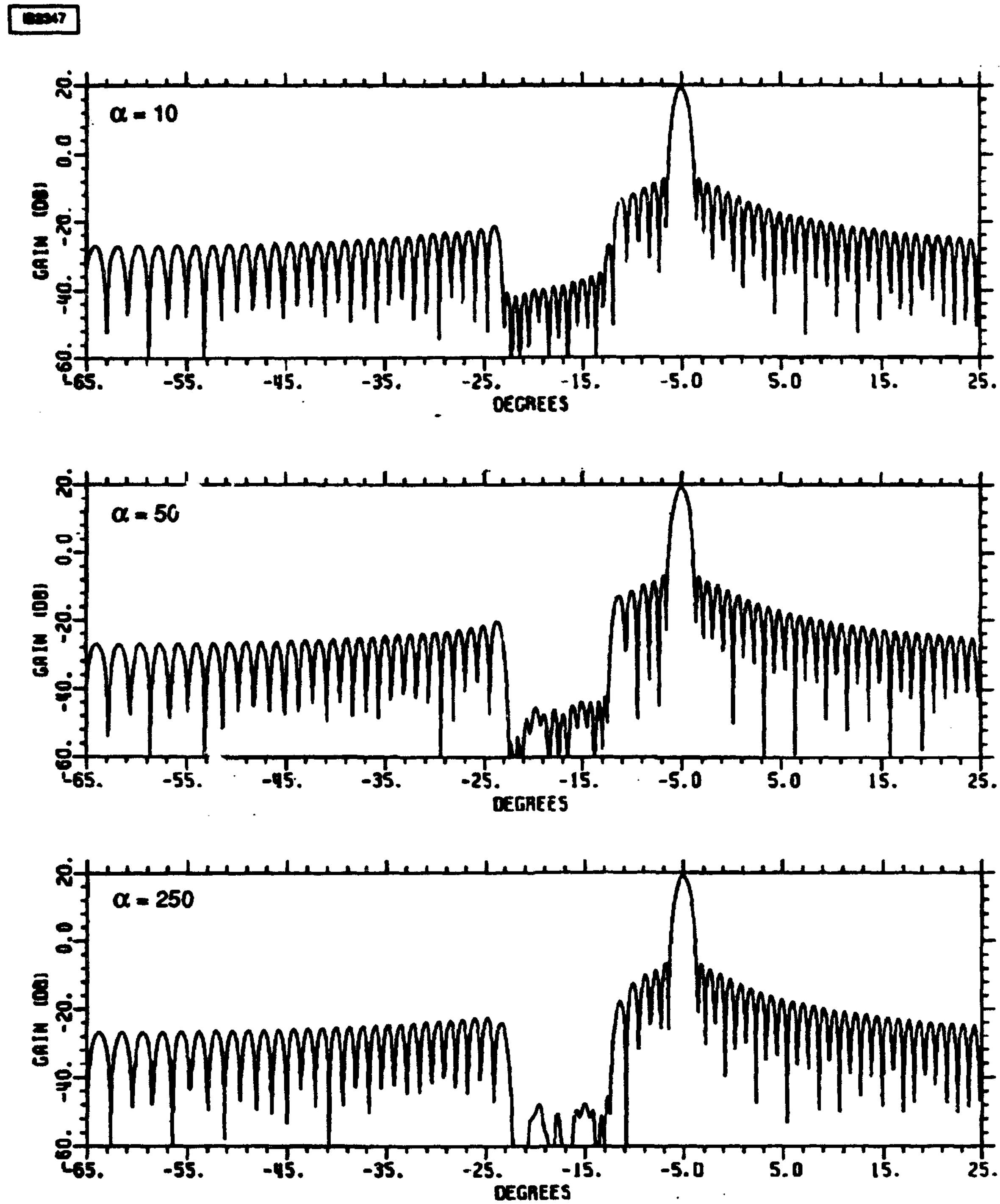

Figure 1. Narrowband (CW) Linear Array Patterns Designed With 10° Notch and Three Different Values of Emphasis Factor (α)

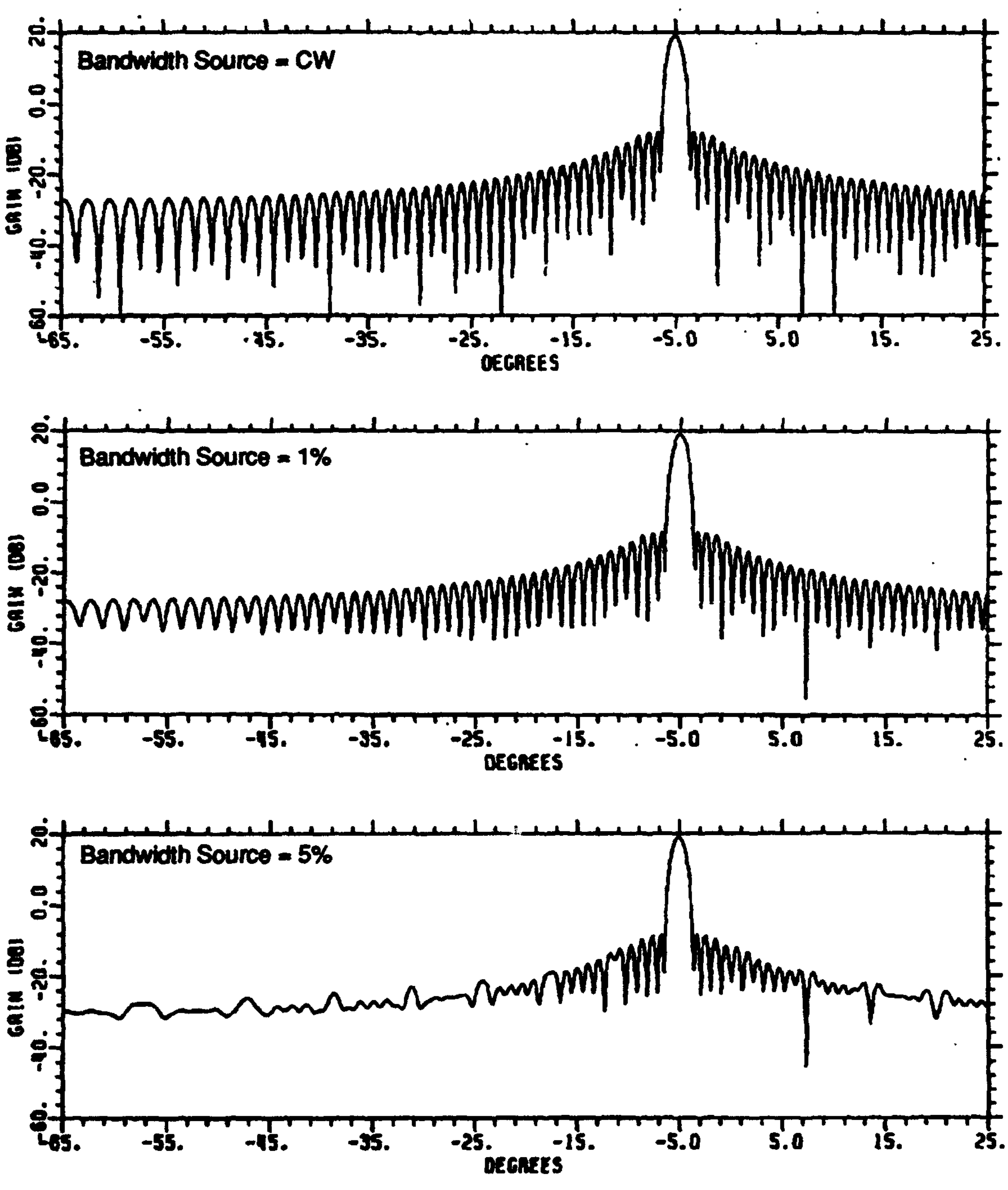

Figure 2. Linear Array Patterns Generated Using Narrowband (CW) Weights and Three Different Bandwidths of Far-Field Noise Source

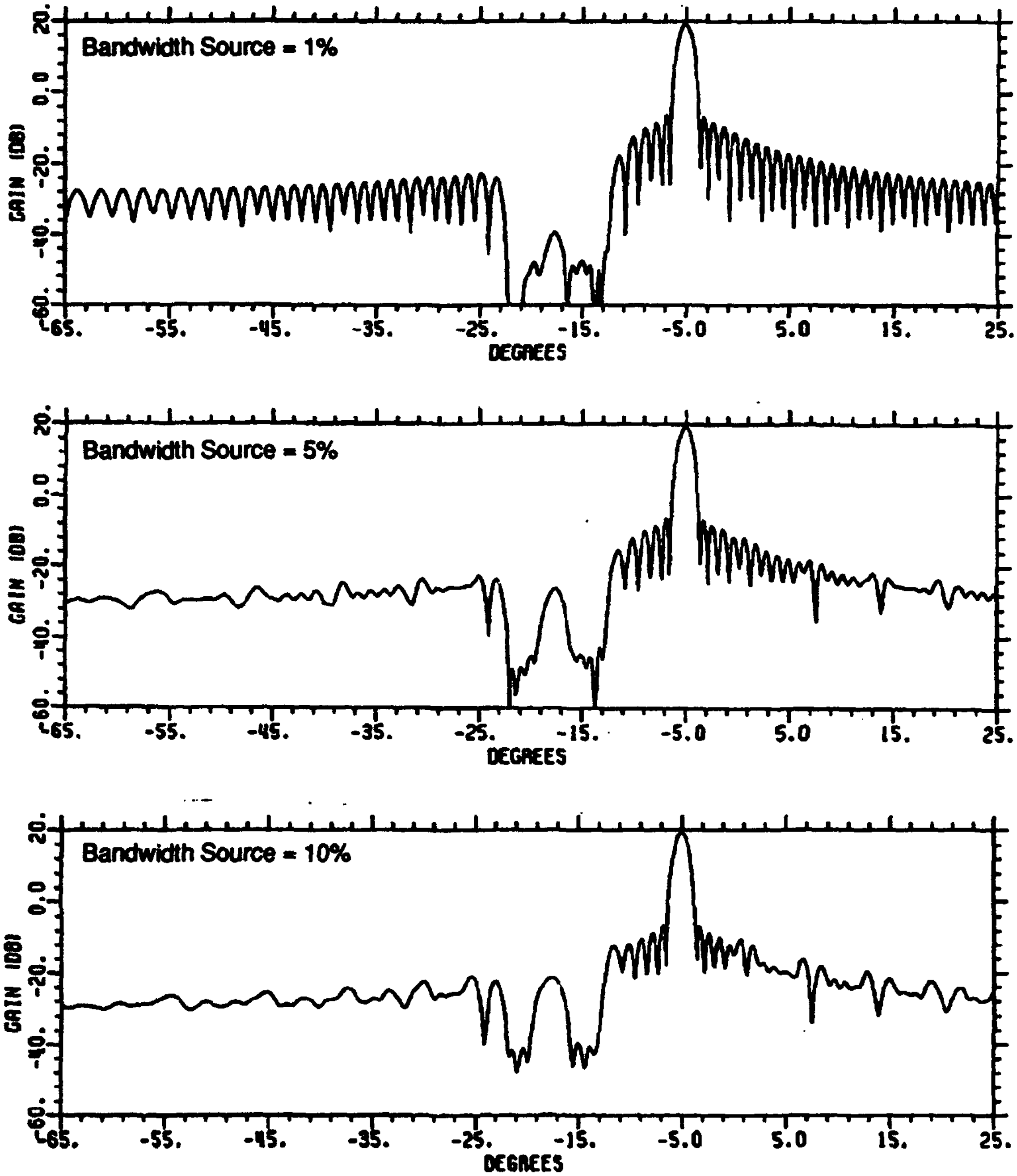

Figure 3. Linear Array Patterns With 10° Notch ($\alpha = 250$) Designed Using Narrowband (CW) Weights and Three Different Bandwidths of Far-Field Noise Source

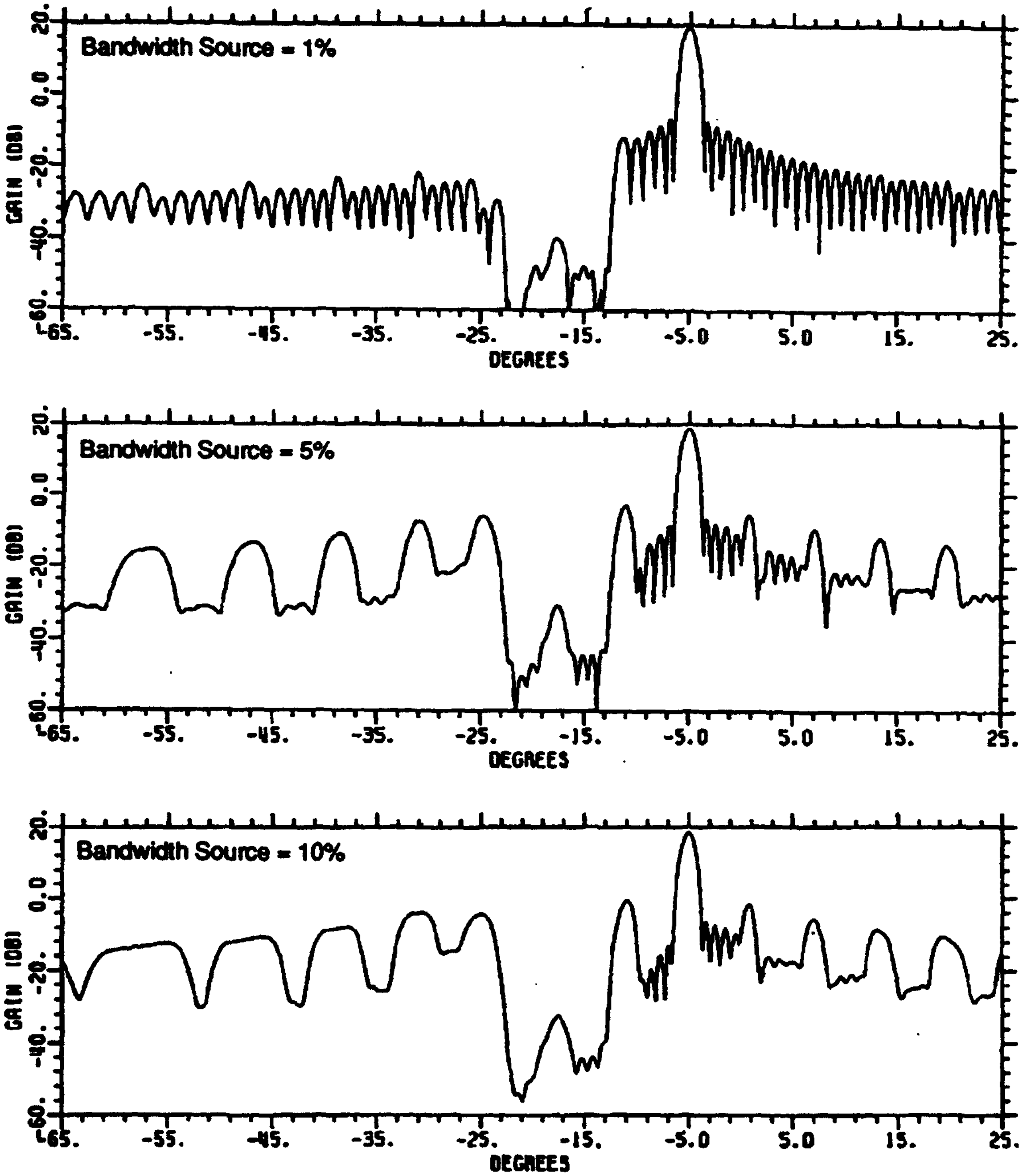

Figure 4. Linear Array Patterns With 10° Notch ($\alpha = 250$) Designed Using Different Sets of Wideband Weights Matched to Bandwidth of Far-Field Noise Source

Using a small value for the emphasis parameter α preserves pattern integrity, but produces a shallow notch. Use of a large value of α degrades the pattern shape, but produces a deep notch. Notch depth also affects the loss in gain at the peak of the mainlobe. Figures 5 and 6 plot the average cancellation ratio in the notched sector and loss in gain at the peak of the beam versus α for one narrowband (fractional bandwidth 0.1 percent) and one wideband (fractional bandwidth 10 percent) design. The 10° notched sector was removed from the mainlobe in Figure 5, but was adjacent to it in Figure 6. The cancellation ratio was defined to be

$$CR = \frac{w_o^+ H w_o}{w^+ H w} \tag{14}$$

where

w^+Hw = average power received within the notched region

$w_o^+Hw_o$ = average power received within the notched region using Taylor weights (no notch)

The loss in peak gain (L) when a deterministic (versus stochastic) signal is received at the peak of the beam (θ_o) was defined to be (using notation defined in Equation (A-2))

$$L = \frac{1}{\gamma MN2\beta} \left| \int_{\omega_o-\beta}^{\omega_o+\beta} g(\mu_o, \omega) d\omega \right|^2 \tag{14}$$

where μ_o equals $\sin\theta_o$ and γ is a normalization factor given by

$$\gamma = \sum_{p=1}^{M} \sum_{r=1}^{N} \left| w_{pr} \right|^2 \tag{15}$$

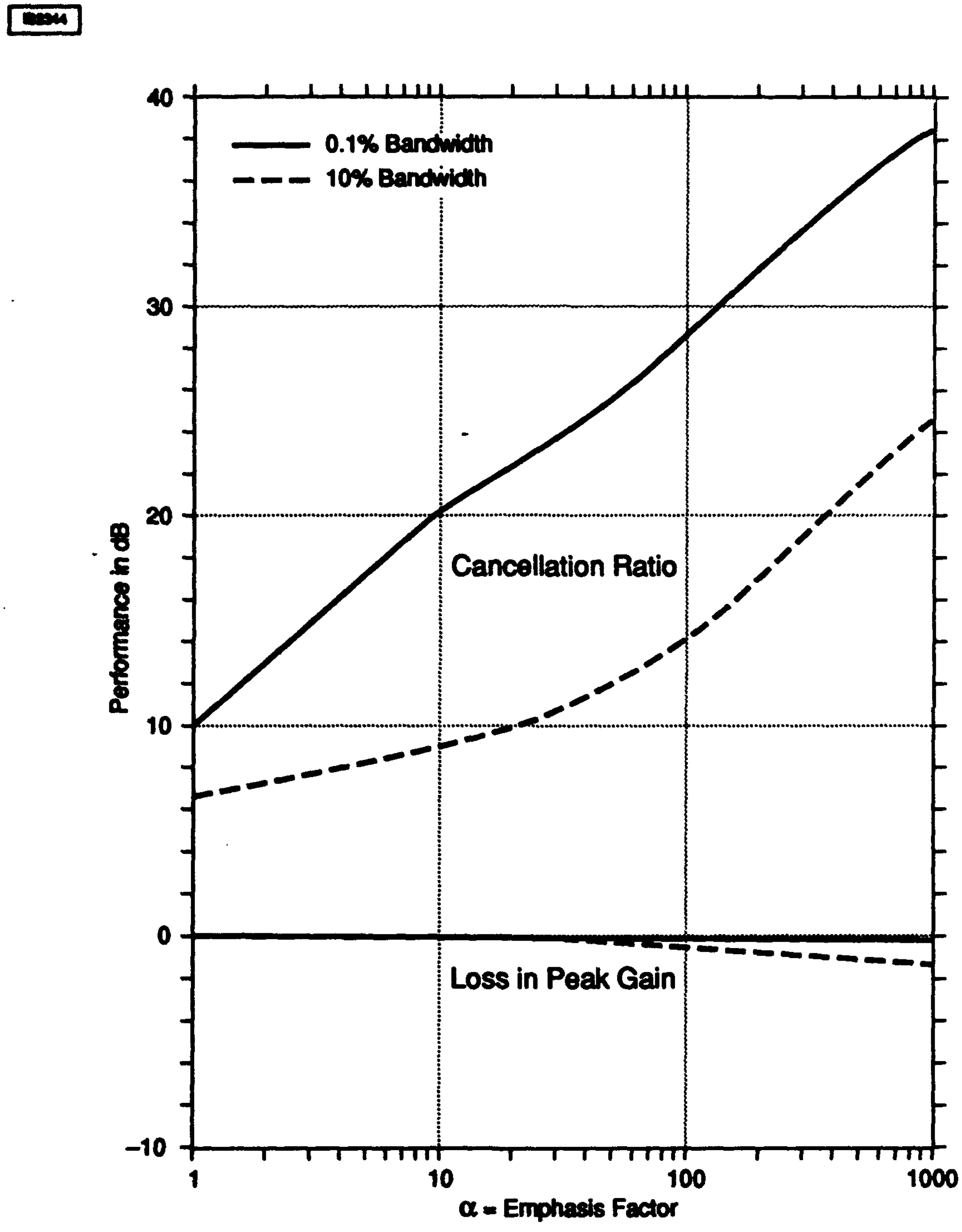

Figure 5. Cancellation Ratio in Notched Sector and Loss in Peak Gain Versus Emphasis Factor for 10° Notch Centered 12.5° From Beam Pointing Direction

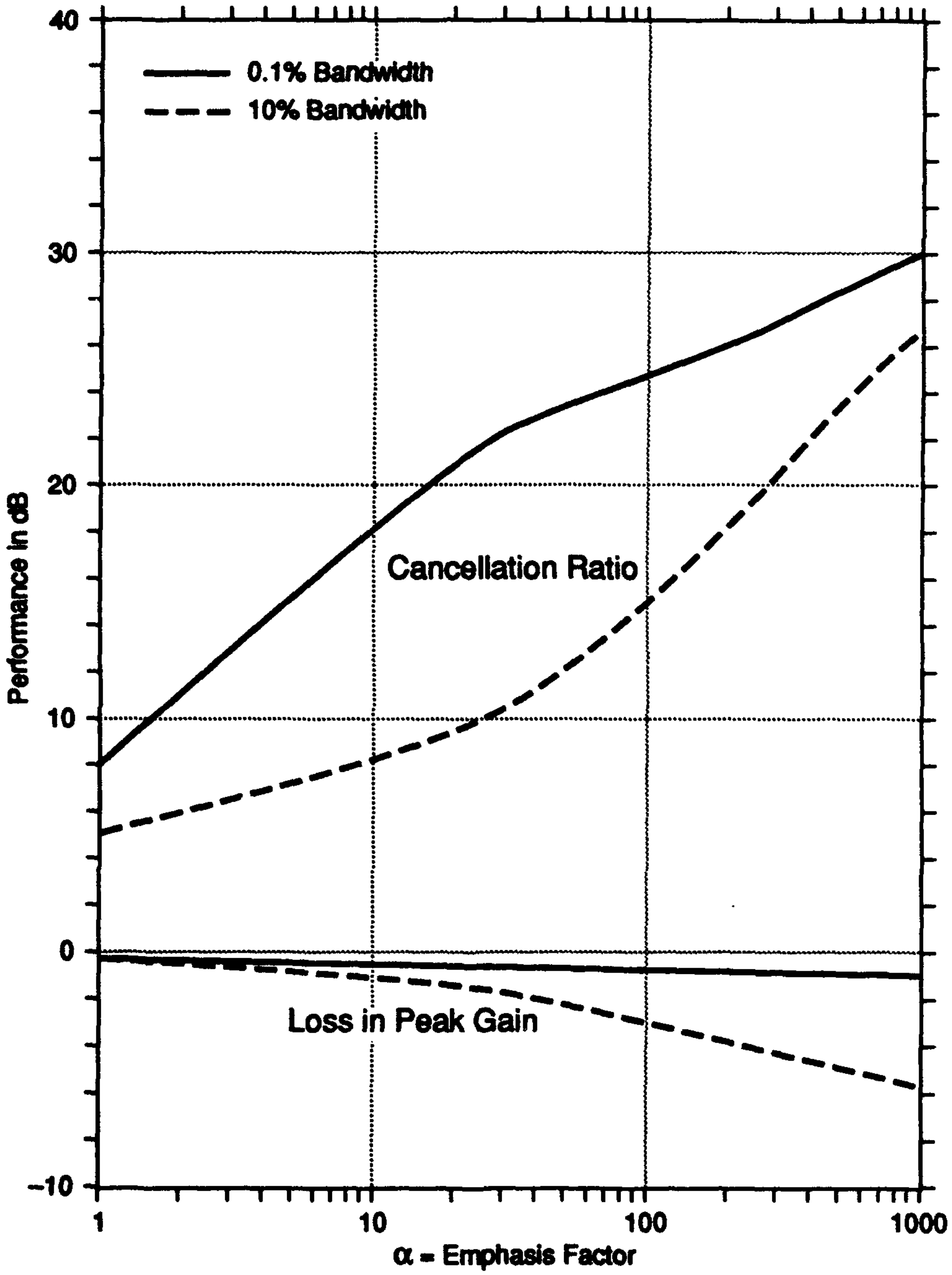

Figure 6. Cancellation Ratio in Notched Sector and Loss in Peak Gain Versus Emphasis Factor for 10° Notch Adjacent to Mainlobe 7° From Beam Pointing Direction

If the design bandwidth is large enough to produce a time-bandwidth product in the notched region which approaches or exceeds unity, the signals at the output of the subarrays will be mutually uncorrelated, and the only way a wideband synthesis procedure can create a notch is to force it independently into each subarray. The subarray notches will then attenuate subarray errors in the notched region. The depth of many notches in wideband patterns will, therefore, be limited by element errors and not by subarray errors. There will be a region surrounding the mainlobe, however, in time steered arrays, where the time-bandwidth product is less than unity and a wideband signal will still be partially correlated at the subarray outputs. Thus the wideband synthesis procedure may not force totally independent notches in the subarray patterns in the near-in sidelobes.

SECTION 5

SUMMARY

Mathematical relationships were derived for the power out of an antenna due to both stochastic and deterministic wideband input signals. The relationships were used to derive definitions for broadband antenna patterns. A wideband synthesis technique for designing notched antenna patterns on phase, time, or phase time steered arrays was derived by modifying an existing narrowband procedure. The technique uses a single parameter to control the depth of the notch. The parameter also affects the loss in gain at the peak of the beam. A number of notched pattern designs were presented to demonstrate use of the new technique. The new approach was shown to alleviate problems associated with the notches filling in when using large fractional bandwidths on phase-time steered arrays.

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APPENDIX A

DERIVATION OF POWER IN NULLED SECTOR

The average power in the angular sector centered at μ_c with half width ε over the frequency range centered at ω_0 with half width β is given by (using the notation defined in Section 3).

$$\langle P\rangle = \frac{1}{4\varepsilon\beta}\int_{\omega_0-\beta}^{\omega_0+\beta}\int_{\mu_c-\varepsilon}^{\mu_c+\varepsilon}|g(\mu,\omega)|^2 d\mu d\omega \qquad \text{(A-1)}$$

where

$$g(\mu,\omega) = \sum_{p=1}^{M} b_p e^{j\omega\tau_p}\sum_{r=1}^{N} w_{pr} e^{j\omega d_{pr}\mu/c}, \qquad \text{(A-2)}$$

b_p = complex weight on p^{th} subarray

τ_p = time delay on p^{th} subarray

w_{pr} = complex weight on r^{th} element in p^{th} subarray (including phase shifter setting)

d_{pr} = position r^{th} element in p^{th} subarray with respect to edge of full aperture

M, N = number of subarrays (M), and number elements per subarray (N)

c = speed of light

Substituting (A-2) into (A-1)

$$\langle P\rangle = \frac{1}{4\varepsilon\beta}\sum_{p=1}^{M}\sum_{q=1}^{M}\sum_{r=1}^{N}\sum_{s=1}^{N} b_p b_q^* w_{pr} w_{qs}^* \int_{\omega_0-\beta}^{\omega_0+\beta}\int_{\mu_c-\varepsilon}^{\mu_c+\varepsilon} e^{j\omega\left[\left(d_{pr}-d_{qs}\right)\mu/c+\left(\tau_p-\tau_q\right)\right]} d\mu d\omega . \qquad \text{(A-3)}$$

Performing the integration over ω gives

$$\langle P\rangle = \frac{1}{4\varepsilon\beta} = \frac{1}{4\varepsilon\beta}\sum_{p=1}^{M}\sum_{q=1}^{M}\sum_{r=1}^{N}\sum_{s=1}^{N} b_p b_q^* w_{pr} w_{qs}^* \int_{\mu_c-\varepsilon}^{\mu_c+\varepsilon} \frac{e^{j(\omega_0+\beta)A_{pqrs}(\mu)} - e^{j(\omega_0-\beta)A_{pqrs}(\mu)}}{jA_{pqrs}(\mu)} d\mu \,, \tag{A-4}$$

where the function variable $A_{pqrs}(\mu)$ is given by

$$A_{pqrs}(\mu) = \left[\left(d_{pr} - d_{qs}\right)\mu / c + \left(\tau_p - \tau_q\right)\right]. \tag{A-5}$$

For notational simplicity, we will denote $A_{pqrs}(\mu)$ by A. Changing the integration variable from μ to A, Equation (A-4) becomes

$$\langle P\rangle = \frac{1}{4j\varepsilon\beta}\sum_{p=1}^{M}\sum_{q=1}^{M}\sum_{r=1}^{N}\sum_{s=1}^{N} \frac{b_p b_q^* w_{pr} w_{qs}^*}{\left(d_{pr} - d_{qs}\right)/c}\left[\int_{A-}^{A+} \frac{e^{j(\omega_0+\beta)A}}{A} dA - \int_{A-}^{A+} \frac{e^{j(\omega_0-\beta)A}}{A} dA\right], \tag{A-6}$$

where

$$A_{\pm} = \left[\left(d_{pr} - d_{qs}\right)\left(\mu_c \pm \varepsilon\right)/c + \left(\tau_p - \tau_q\right)\right]. \tag{A-7}$$

The integrals in the above expression for the average power can be expressed as cosine and sine integrals which can be evaluated numerically. Thus, we can write the integral terms inside the braces as

$$\begin{aligned}[\cdots] = {} & \mathrm{Ci}\{(\omega_0+\beta)A_+\} - \mathrm{Ci}\{(\omega_0+\beta)A_-\} + j\mathrm{Si}\{(\omega_0+\beta)A_+\} - j\mathrm{Si}\{(\omega_0+\beta)A_-\} \\ & - \mathrm{Ci}\{(\omega_0-\beta)A_+\} + \mathrm{Ci}\{(\omega_0-\beta)A_-\} - j\mathrm{Si}\{(\omega_0-\beta)A_+\} + j\mathrm{Si}\{(\omega_0-\beta)A_-\} \,,\end{aligned} \tag{A-8}$$

where

$$\mathrm{Ci}\{(\omega_0 \pm \beta)A_\pm\} = -\int_{A_\pm}^{\infty} \frac{\cos((\omega_0 \pm \beta)A)}{A}\, dA\,, \tag{A-9}$$

and

$$\mathrm{Si}\{(\omega_0 \pm \beta)A_\pm\} = \int_{0}^{A_\pm} \frac{\sin((\omega_0 \pm \beta)A)}{A}\, dA\,. \tag{A-10}$$

Recall that the design under consideration is adaptable at the element level. With this in mind, the expression for the average power $\langle P\rangle$ can be written in the matrix form $w^{+}Hw$, where w is the 1 x MN column vector of element excitations, and H is the MN x MN matrix given by

$$H_{pr,qs} = \frac{b_p b_q^*}{4j\varepsilon\beta\left(d_{pr} - d_{qs}\right)/c} \begin{Bmatrix} \mathrm{Ci}\{(\omega_0+\beta)A_+\} - \mathrm{Ci}\{(\omega_0+\beta)A_-\} \\ \quad + j\mathrm{Si}\{(\omega_0+\beta)A_+\} - j\mathrm{Si}\{(\omega_0+\beta)A_-\} \\ -\mathrm{Ci}\{(\omega_0-\beta)A_+\} + \mathrm{Ci}\{(\omega_0-\beta)A_-\} \\ \quad - j\mathrm{Si}\{(\omega_0-\beta)A_+\} + j\mathrm{Si}\{(\omega_0-\beta)A_-\} \end{Bmatrix}. \tag{A-11}$$

The above expression is valid for the off-diagonal terms of H. Setting r = s and p = q in (A-11), the diagonal terms are observed to be $|b_p|^2$.

It is also noteworthy that if we set β equal to zero and integrate (A-1) only over μ the analysis simplifies to

$$\langle P\rangle = \sum_{p=1}^{M}\sum_{q=1}^{M}\sum_{r=1}^{N}\sum_{s=1}^{N} w_{pr} w_{qs}^* e^{j\omega_0\left[(\tau_p-\tau_q)+(d_{pr}-d_{qs})\mu_c/c\right]} \mathrm{sinc}\left(\frac{\omega_0\left(d_{pr}-d_{qs}\right)\varepsilon}{c}\right)$$

$$= w^{+}H_0 w\,. \tag{A-12}$$